MTH150 Project 4

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1 Chapter 4, Exponential and Logarithmic Functions

1.1 4.1 Exponential Functions

Question 7, Page 263
A population numbers 11,000 organisms initially and it grows by 8.5 percent each year. Write an exponential model for the population.
For this problem, we will be using $f(x) = a(1 + r)^x$
a= initial/starting point, $a = 11,000$
r =percent growth/decay rate, $r = 8.5$
What the question is asking is to write the model, there is no need to solve for anything, therefore, after plugging in all the values, the model would look like $f(x) = 11,000(1 + 8.5)^x$
After simplifying, the model would be $f(x) = 11,0000(1.085)^x$

Question 13, Page 264
Find a formula passing through the points $(0, 6)$ and $(3, 750)$.

To start, we want to use the formula $f(x) = ab^x$. From here, it is needed to substitute some of our values into the equation. Using the coordinate $(0, 6)$, we can find our a value. Because x is zero, the equation will look like $f(0) = ab^0$. We can conclude that $a = 6$ (another way to solve is when looking at the coordinates, the y value will be equal to a).

So far, the question has he set up of $f(x) = 6b^x$, however we need to solve for b. Now we will use our other coordinate, $(3, 750)$. Plug in the values into the appropriate locations. $750 = 6b^3$.
$b^3 = \frac{750}{6} = 125$
$b = \sqrt[3]{125} = 5$
$f(x) = 6(5)^x$

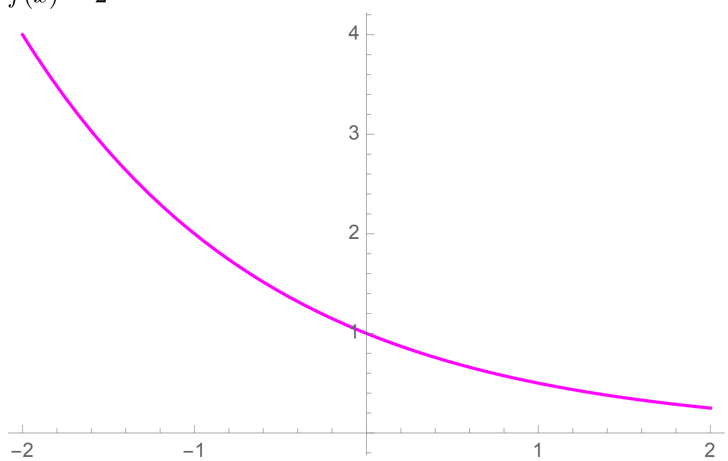
Question 23, Page 264
A radioactive substance decays exponentially. A scientist begins with $100mg$ of a radioactive substance. After 35 hours, $50mg$ of the substance remains. How many mg will remain after 54 hours?

Just like the last problem, we will be using $f(x) = ab^x$ to solve. From what is given to us in the word problem, we have enough information to solve. After plugging in the knowns, our equation will look like $50 = 100b^35$.
$b^35 = \frac{50}{100} = .5$
$\sqrt[35]{.5} = .98039$
$f(x) = 100(.98039)^54 = 34.319mg$
$34.319mg$ remains after 54 hours.

For this section, Exponential functions, I thought that it was quick to finish and easy to understand. When I was doing he work for the questions I didn't have to go back and re-read any of my notes on how to solve for the equations, and I completed everything in less than 20 minutes. I personally didn't have a problem with logarithms even in high school, however I did think that this chapter would have been a lot harder to complete, but that never became a problem for this section.

1.2 4.2 Graphs of Exponential Functions

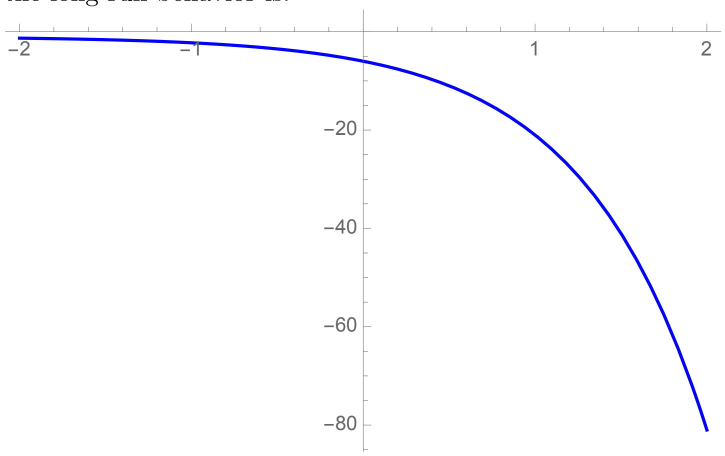
Question 11, Page 275
Sketch a Graph of each of the following transformations of 2^x
$f(x) = 2^{-x}$



Question 17, Page 275
Shifting with the graph of $f(x) = 4^x$ find a formula for the function that results from:
Shifting $f(x)$ 4 unites upwards.
$f(x) = 4^x + 4$

Question 23, Page 276
Describe long run approaches $x \to \infty$ and $x \to -\infty$ for each function.

$f(x) = -5(4^x) - 1$
For this question, we are working with the formula $f(x) = ab^x$. $a = -5$ $b = 4$
The a value is the vertical intercept of the graph, and the b value determines the rate at which the graph grows.
Below is the graph for the function $f(x) = -5(4^x)$ to get a better look at what the long run behavior is.



From this, we can see that as $x \to -\infty$, $f(x) \to -1$ and as $x \to \infty$, $f(x) \to -\infty$.

For this section, Graphs of Exponential Functions, I completed the questions very easily and quickly. Unlike section 4.1, I was required to sketch graphs, which I used Mathematica for, and found it as a very easy way to find the long run behavior for question 23 on page 276. I understand how to solve for long run behavior, however having a visual of what the graph looks like makes solving a lot easier for me. For the other two questions, I found them easy to go through. The only question that did not require a graph was question was question 17, and I thought that that question was easier than the rest because it was simply asking for a transformation. Just like with section 4.1, I flew hrough this section and had no issues with solving.

1.3 4.3 Logarithmic Functions

Question 1, Page 287
Rewrite each equation in exponential form.
$\log_4(q) = m$
We need to re-write the equation before we can put it into exponential form, which would look like $m \log_4 = \sqrt{q}$, next we can solve $4^m = 9$. We received this answer by having the original function set up like $Log_b(c) = a$, and then transforming the function to look like $b^a = c$.

Question 9, Page 287

rewrite each equation in log form.
$4x = y$
$\log_4(y) = x$

Question 17, Page 287
Solve for x.
$\log_3(x) = 2$
$x = 3^2$ $x = 9$
Question 41, Page 287
Solve each equation for the variable.
$5^x = 14$
$\log_5(14) = x$

Question 44, Page 287
Solve each equation for the variable.
$3^x = \frac{1}{4}$
$\log_3(\frac{1}{4}) = x$

Question 65, Page 288
The population of Kenya was 39.8 million in 2009 and has been growing about 2.6 percent each year. If this trend continues, when will the population hit 45 million?
For this problem we will be using the formula $f(x) = a(1+r)^x$. We know that $a = 39.8$, and the growing rate, r, is $r = 2.6$. Plug in the knowns into the equation. $f(x) = 39.8(1 + 2.6)^t$.
From here, I feel like I might have done the problem differently than some others. What I did was plug in years since 2009 and checked to see when the population would finally reach over 45 million people. After this, I got that it took 5 years until it reached 45 million.
$f(5) = 39.8(1.026)^5$= 45,250,134 million.

For this section, I also felt like I understood what I had to do in order to solve for each equation. I of course read the section in the book and then got to work. I felt like this section was somewhat repetitive, but that was only because we had to keep changing the form that the log was in. I personally wouldn't say that I struggled on any question in particular, however if I had too chose I would say question 65, and I say that simply because it is a word problem, and in order to solve, you need to dig through the problem to know your data. Besides this, this section was easy like the rest for me.

1.4 4.4 Logarithmic Properties

Question 1, Page 298
Simplify to a single log, using log properties.
$\log_3(28) - \log_3(7)$

For this problem, to get a single Log value, we will be dividing the c values. After this, we are left with $\log_3(4)$.

Question 17, Page 298
Use log properties to expand each expression.
$\log(\frac{x^15y^13}{z^19})$
From this we will get $15\log(x) + 13\log(y) - 19\log(z)$.

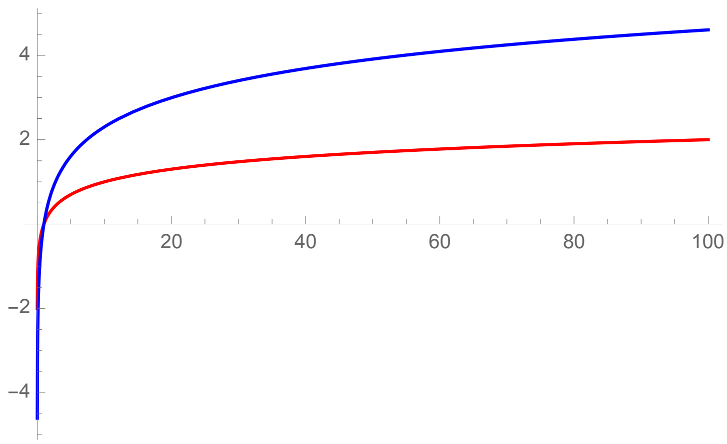
For this section, I thought that it was really short and was kind of surprised because out of the problems I have solved so far, they were more difficult than the rest. I felt like more questions in this section should have been assigned simply for students to get a better grasp of what this section was asking them to do. Besides this, I still solved the problems relatively quickly, even though I did find them more difficult than the previous sections.

1.5 4.5 Graphs of Logarithmic Functions

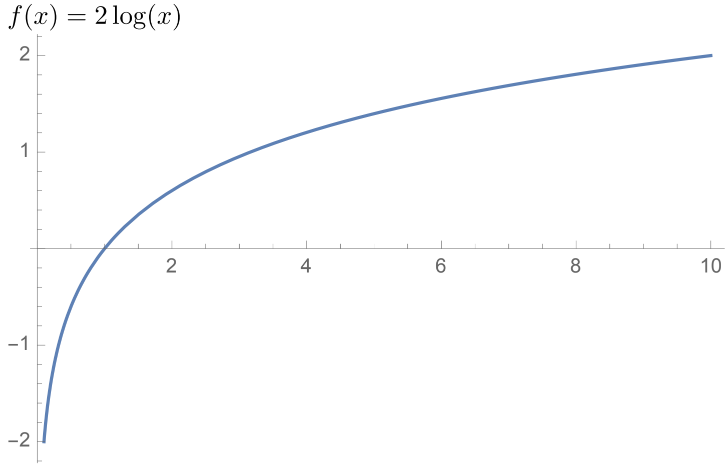
Question 1, Page 306
For each function, find domain and vertical symptotes.
$f(x) = \log(x - 5)$
The domain is $x > 5$, and the vertical asymptote is 5.

Question 5, Page 306
$f(x) = \log(3x + 1)$
The domain would be $3x > 1$, however that is not simplified. After simplification, the domain would be $\frac{1}{3}x > 1$. The vertical asymptote would be located at $\frac{1}{3}$.

Question 9, Page 306
Sketch a graph of each pair of functions.
$f(x) = \log(x)$, and $g(x) = In(x)$



Question 11, Page 306
Sketch each transformation
$f(x) = 2\log(x)$



For this section, it was based on graphing Log functions. I thought that because of this it would be easy, I could just plug in what I need into Mathematica and then insert the graph into the document. When I went to do that though, I didn't know how to correctly write the function into Mathematica for a graph to appear. I originally thought I needed to write out the log function just like a linear function (i.e. In Mathematica, you first would need to write out the function ($f(x)$:=(insert a linear function here)), and then would Write $Plot[f[x], x, 10, -10, PlotStyle \rightarrow Magenta]$ and then there would be a graph of your function. However, as you helped me I found out that when graphing logarithms, its quite different. There is no need to write the equation out before writing plot, you can just say the equation in the command, and in addition to what coordinates you want the frame of view to be, before that you would write $[10, x]$ since log is log to base 10. However, after your help I understood what

I was doing wrong, and thought that the command was obvious (even though that wasn't always the case).

1.6 4.6 Exponential and Logarithmic Models

Question 1, Page 322 You go to the doctor and he injects you with 13 milligrams of radioactive dye. After 12 minutes, 4.75 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm whenever more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived and the amount of dye decays exponentially?
The formula that we will be using is $m(t) = ab^t$
$a = 13$, $m(t) = 13b^t$
$\frac{4.75}{13} = \frac{13b^12}{13}$=.3653 which means $b^12 = .3653$
After this we need to get b by itself, which would be set up like $\sqrt[12]{.3653} = .9195$
$b = .9195$
Now that we know what our b value is, we can solve for the remainder of the problem. 2 is the initial value, so once that is plugged in, the equation will look like $2 = 13(.9195)^t$.
Now, we need to divide both sides by 13.
$\frac{2}{13} = .9195^t$
It is now time to solve for how long the doctors appointment was, we can do this by using logarithms.
$\log(\frac{2}{13}) = \log((.9195)^t)$
$\log(\frac{2}{13}) = t\log(.9195)$ $t = \frac{\log\frac{2}{13}}{\log(.9195)}$=22.3 minutes

Question 3, Page 322
The half-life of Radium-226 is 1590 years. If a sample initially contains 200 mg, how many milligrams will remain after 1000 years?
We know that $t = 1590$, and since we are dealing with half-life, the a value will be half of what normally would be. Therefore, the equation would be set up as $.5a = ab^1590$. From here, divide both sides by a
$.5 = b^1590 = b = (.5)\frac{1}{1590}$
$b = .9995$
We are also aware that $a = 200$. With that being said, we are able to plug in all of our values to see how many milligrams will remain.
$h(1000) = 200(.9995)^{1000}$=121.29 years

Question 9, Page 322 A wooden artifact from an archeological dig contains 60 percent of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)
For this equation, we will be using the formula $.5a = ae^{rt}$. We know that our t value will be 5730, so from there we can work out to find the rate of decay.
$In(\frac{1}{2} = In(er5730$

$\frac{In\frac{1}{2}}{5730} = \frac{5730r}{5730}$
$r = \frac{In\frac{1}{2}}{5730} = -.000121$
Now that the rate is known, we can solve for how old the artifact is.
$Q(t) = a^{e-.000121t}$
$Q(t) =$ percentage of carbon-14, $Q(t) = .60a$
$.60a = a^{e-.000121t}$
$In(.60) = In(e^{-000121t}$
$In(.60) = -.000121$
$t = \frac{In(.60)}{-.000121} = 422.169$ years.

Question 10, Page 323
A wooden artifact from an archeological dig contains 15 percent of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)
Luckily, in the last problem carbon-14 was also being used, meaning that the rate of decay will be the same. Therefore, the equation would look like $t = \frac{In(.15)}{-.000121}$=1,567,677 years.

So far, this has been the most difficult section for me to complete besides 4.5, however that was due to the graphing. In this section, for each question many steps had to be made in order to get the final answer that was needed. From all the questions in this section I would say that question 9 was the most difficult, simply because it was the first question that I had to solve and didn't have any previous knowledge on how to solve for. A lot of trial and error was done while I was completing this section. In order to make sure that I was correct, I would check the solution manual to see if what I had done made sense, and if it didn't, I would go back and re-work it. Sometimes even the solution manual wouldn't make sense to me, so I went back, looked at the formulas at hand, and the known values, and broke them down so I would understand the basics of the question was before I tried to tackle all of it. I saw that doing that made things a lot easier for me and I plan on using that in the future when I have difficulties solving a problem.

1.7 4.7 Fitting Exponential Models to Data

Question 9, Page 335
Use regression to find an exponential function that best fits the data given.

1	1125
2	1495
3	2310
4	3294
5	4650
6	6361

To find the exponential function, we will be start by dividing each value by one

another.
$\frac{1495}{1125} = 1.328$
$\frac{2310}{1495} = 1.545$
$\frac{3294}{2310} = 1.425$
$\frac{4650}{3294} = 1.411$
$\frac{6361}{4650} = 1.367$

Now, we find the average which would be 1.4152.
The formula we will be working with is $f(x) = a(b)^x$. Our average is equal to our b value, and we can use our $(1, 1125)$ coordinate for our x and y value. Therefore,our function looks like $1125 = a(1.4152)^1$. Divide by 1125 by 1.4125, to get our a value which is 794.94, meaning that our answer will be $f(x) = 794.94(1.4152)^x$.

Question 10, Page 335

1	643
2	829
3	920
4	1073
5	1330
6	1631

To find the exponential function, we will be start by dividing each value by one another.
$\frac{829}{643} = 1.2892$
$\frac{920}{829} = 1.1097$
$\frac{1073}{920} = 1.1663$
$\frac{1330}{1073} = 1.2395$
$\frac{1631}{1330} = 1.2263$
Now, we find the average which would be 1.2062.
The formula we will be working with is $f(x) = a(b)^x$. Our average is equal to our b value, and we can use our $(1, 643)$ coordinate for our x and y value. Therefore,our function looks like $643 = a(1.2062)^1$. Divide 643 by 1.2062, to get our a value which is 533.07, meaning that our answer will be $f(x) = 533.07(1.2062)^x$.

Question 11, Page 335

1	555
2	383
3	307
4	210
5	158
6	122

Now we find our average which is .73986.
The formula we will be working with is $f(x) = a(b)^x$. Our average is equal to our b value, and we can use our $(1, 555)$ coordinate for our x and y value. Therefore,our function looks like $555 = a(.73986)^1$. Divide 555 by .73986 to get our a value, which is 750.141, meaning that our answer will be $f(x) = 750.141(.73986)^x$.

Question 12, Page 335

1	699
2	701
3	695
4	668
5	683
6	712

To find our average, we will be dividing each number by one another to get our average which is 1.0037. The formula we will be working with is $f(x) = a(b)^x$. Our average is equal to our b value, and we can use our $(1, 699)$ coordinate for our x and y value. Therefore,our function looks like $699 = a(1.0037)^1$. Divide 1.0037 by 699 to get 696.42. Therefore, the answer of this question will be $f(x) = 696.42(1.0037)^x$.

For this section, I was originally struggling trying to understand how to solve for the problems. I didn't know how to set up the question and thought that it was more complicated than it actually was. Once I did the first question, the rest were easy to do.

For chapter 4, Overall I thought it was easy and I flew through a majority of the questions in the sections. The two sections I struggled the most with were 4.5 and 4.7. In 4.5 I got stuck on graphing the functions in Mathematica, however you were able to help me by sending me PDFs on how properly graph logarithms. In 4.7, I kept putting it off, because I didn't understand what to do for it. However once I got the hang of it I flew through the questions. I thought that this chapter was a lot easier than some others that we had to do.