MTH150 Project 6

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1 Chapter 6, Periodic Functions

1.1 6.1: Sinusoidal Graphs

For each of the following equations, find the amplitude, period, horizontal shift, and midline.
Question 11, Page 410
$y = 3\sin(8(x+4)) + 5$

To start, the formula setup for this equation is $y = A\sin(B(t)) + k$. Lets begin by breaking this down. The A value, is equal to the amplitude of the equation, meaning that $A = 3$. B is equal to he horizontal stretch or compression of the function, in this case $B = 8$. Even though it does not ask for the horizontal stretch/compression exactly, finding the value of he period requires knowing the value of B. To find the period of the function, he equation looks like $\frac{2\pi}{B}$, meaning that for this function, the period is equal to $\frac{\pi}{4}$. Moving on to the horizontal shift, the value of the horizontal shift is described as the value of t. Therefore, in this equation, $t = x + 4$, and the horizontal shift is 4 to the left. Lastly, the midline is equal to the value of k, meaning that $k = y = 5$.

Question 13
$y = 2\sin(3x - 21) + 4$
$A = 2$
$B = 3$
period=$\frac{2\pi}{3}$
horizontal shift= 21 to the right
midline=$y = 4$

Question 21, Page 411
Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature is 50 degrees at midnight and the high and low temperature during the day are 57 and 43 degrees, respectively. Assuming t is the number of hours since midnight, find a function for the temperature, D, in terms of t.

First, we need to find the midline for for this problem. Since we know the high and low temperatures during the day are 57 and 43, we can add them together and divide by 2 to find the midline. $\frac{57+43}{2} = 50$ The midline of this problem is 50.

Secondly, we need to find the amplitude. The amplitude will be the difference between the high and low point, meaning that the amplitude will be 7. We also know that 24 hours are in a day, meaning that our equation will be, $D = 50 - 7\sin(\frac{2\pi}{24}t)$. This can be brought down to, $D = 50 - 7\sin(\frac{\pi}{12})$.

Question 23
A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function $h(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.
a) Find the amplitude, midline, and period of $h(t)$.

the period will be ten minutes since the length of the ride is 10 minutes long. The midline of the diameter of he wheel will be the sum of 24 and 1 divided by 2, $\frac{24+1}{2} = 12.5$. The amplitude will be the sum of 26 and 1 divided by 2, $\frac{26+1}{2} = 13.5$.
b) Find a formula for the height function $h(t)$.

The setup of this formula will be the same as question 11 and 13, and we already know everything that is needed to write the the formula. $h(t) = 12.5\cos(\frac{2\pi}{10}t) + 13.5$. This can be brought down to, $h(t) = 12.5\cos(\frac{\pi}{5}t) + 13.5$.
c) How high are you off the ground after 5 minutes?

This can be solved rather easily since we know the length of the ride is 10 minutes. half of 10, is 5, meaning that that is halfway through the duration of the ride, or half a rotation. Since at 10 minutes, the cart will be back at the lowest point of the ride, we know that at 5 minutes it must be at the highest point. This means that at 5 minutes, you would be 26 meters of the ground in your cart.

For this section, Sinusoidal Graphs, I found the concepts very easy to understand and was able to solve all the problems. The question I had the most trouble with was question 23 part a, because I was having trouble understanding the difference between the amplitude and the midline of the diameter of the wheel for this particular problem. However, To make sure I was doing everything correctly I used the solution manual to check my work. Besides this part of question 23, I found 6.1 easy to work through.

1.2 6.2: Graphs of the other Trig Functions

Question 5, Page 419
Find the period and horizontal shift of each of the following functions.
$f(x) = 2\tan(4x - 32)$

the period of the function is equal to $\frac{\pi}{4}$, and the horizontal shift will be 8.

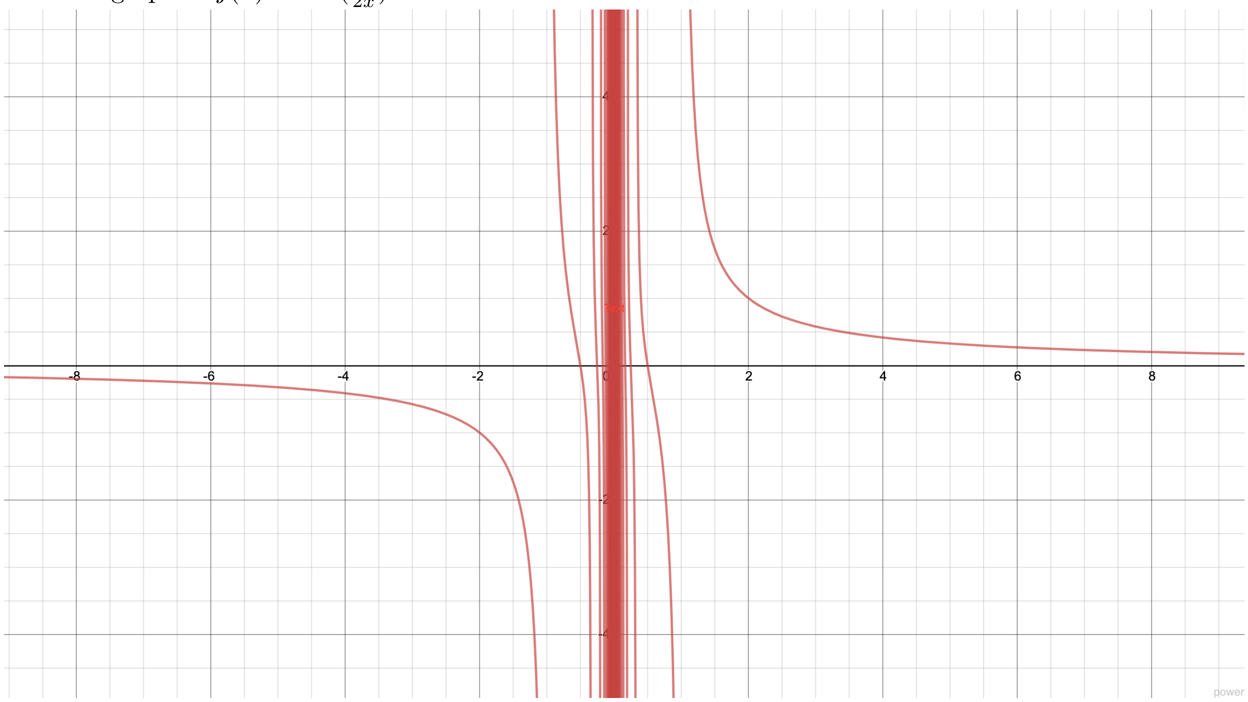
Question 6

$g(x) = 3\tan(6x + 42)$

The period is equal to $\frac{\pi}{6}$, and the horizontal shift is equal to 7.

Question 15, Page 420

Sketch a graph of $j(x) = \tan(\frac{\pi}{2x})$



Question 21, Page 421

If $\tan(x) = -1.5$, find $\tan(-x)$.

Since this is only asking for a negative, $\tan(-x) = 1.5$

Question 23

If $\sec(x) = 2$, find $\sec(-x)$.

Just like the previous question, the answer will be negative compared to $\sec(x)$, meaning that $\sec(-x) = -2$.

Question 27

Simplify each of the following expressions completely.

$\cot(-x)\cos(-x) + \sin(-x)$

To begin, $\cot(-x) = -\frac{\cos(x)}{\sin(x)}$, meaning that when solving we can substitute this into the equation for $\cot(-x)$. From here we have, $(-\frac{\cos(x)}{\sin(x)})(\cos(x)) - sin(x) {=} -(\frac{\cos^2(x)+\sin^2(x)}{\sin(x)}) = -\frac{1}{\sin(x)} = -\csc(x)$

For this section, Graphs of the other Trig Functions, I found tthe first few questions to be very similar to the previous section, and question 15 I used desmos to graph of the function, $f(x) = \tan(\frac{\pi}{2x})$. The hardest question I worked through was the last question, just like the previous section. I found it difficlt because when I was just looking at it, I thought there were a lot of components to it, however, to make it not look as bad imagine sin and cos as x and y. After using this thought process I was able to solve.

1.3 6.3: Inverse Trig Functions

Evaluate the following expressions, giving the answer in radians.
Question 1, Page 429
$\sin^- 1(\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

Question 3
$\sin^- 1(-\frac{1}{2} = \frac{\pi}{6}$

Question 19
Evaluate the following expression: $\sin^- 1(\cos(\frac{\pi}{4})$
$\sin^- 1(\cos(\frac{\pi}{4}) = \sin^- 1(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$

Question 23
$\cos(sin^- 1(\frac{3}{7}))$
$x = \sin^- 1(\frac{3}{7})$
$\sin(x) = \frac{op}{hyp}$
$3^2 + ad^2 = 7^2$
$9 + ad^2 = 49 = ad^2 = 40$
$\sqrt{40} = 2\sqrt{10}$
$\cos(\sin^- 1(\frac{3}{7}) = \cos = \frac{ad}{hyp} = \frac{2\sqrt{10}}{7}$

For this section, Inverse Trig Functions, I found the work easy to do after reading the section in the textbook, I didn't find any particular question hard, but question 23 was lengthy and took me longer to solve than the other three questions did. I felt as though this was a continuation of the first section which is why I had no issues with this section in my opinion. Overall, I flew through this section and didn't have any issues.

1.4 6.4: Solving Trig Equations

Find all solutions on the interval $o \leq \theta < 2\pi$.
Question 1, Page 440

$2\sin(\theta) = -\sqrt{2}$
We know that sin alone would be half of this, therefore we would have $\sin(\theta) = \frac{\sqrt{2}}{2}$ so far. $\theta = \frac{5\pi}{4} + 2\pi(k)$, from this we can find the solutions of our problem, $\theta = \frac{5\pi}{4}$, $\theta = \frac{7\pi}{4}$

Question 3
$2\cos(\theta) = 1$
Just like the last question, we will be dividing 1 by 2 to get cosine alone, therefore, $\cos(\theta) = \frac{1}{2}$
After this, we will have $\theta = \frac{\pi}{3} + 2\pi(k)$. We can now solve for our two solutions, which are $\theta = \frac{\pi}{3}$, $\theta = \frac{5\pi}{3}$

Find all solutions.
Question 9
$2\cos(\theta) = \sqrt{2}$
Isolate cosine by dividing both sides by 2, after this we have $\cos(\theta) = \frac{\sqrt{2}}{2}$.
$\theta = \frac{\pi}{4} + 2\pi(k)$
$\theta = 2\pi(k)$
$\theta = 2\pi - \frac{\pi}{4} + 2\pi(k) = \frac{7\pi}{4}$

Question 11
$2\sin(\theta) = -1$
$\sin(\theta) = -\frac{1}{2}$
$\theta = \frac{7\pi}{6} + 2\pi(k)$, $\theta = \frac{11\pi}{6} + 2\pi(k)$

Question 13
$\sin(3\theta) = \frac{1}{2}$
I solved this problem by using Mathematica, and I will be writing out what I did to solve.
I started by using the command ArcSin$[\frac{1}{2}]$ which gave me $\frac{\pi}{6}$. After doing this I got $\frac{\pi}{6} + \frac{2\pi(k)}{3}$, $\frac{5\pi}{6+2\pi(k)}$. Finally, I had $\frac{\pi}{18} - \frac{2\pi(k)}{3}$, $\frac{5\pi}{18} + \frac{2\pi(k)}{3}$

Question 19
$2\cos(3(\theta)) = \sqrt{2}$
$\cos(3(\theta)) = \frac{\sqrt{2}}{2}$
$3(\theta)) = \frac{5\pi}{4} + 2\pi(k)$
$(\theta) = \frac{5\pi}{12} + \frac{2\pi(k)}{3}$

Question 33
Find the first two positive solutions.
$7\sin(6x) = \frac{2}{7}$
$6x = \sin -1 = (\frac{2}{7})$
$6x = \pi - .28975 = 2.85184$

For this section, I had a very hard time understanding what I had to do to solve, and even after typing it into my project, it still makes little sense to me. I spent a long time trying to understand what to do for this section, and even after emailing you I felt lost. I spent a decent amount of time trying to figure out what to do by using the solution manual, but I still was unable to figure out how to properly work out these problems on my own.

1.5 6.5 Modeling with Trigonometric Functions

Question 7, Page 448
Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature for the day is 63 degrees and the low temperature of 37 degrees occurs at 5 AM. Assuming t is the number of hours since midnight, find an equation for the temperature, D, in terms of t.
The period is 24 since there is 24 hours in a day. The midline can be found by subtracting 37 by 63 and then dividing by 2, $\frac{63-37}{2} = 13$. The amplitude is the sum of 63 and 37, divided by 2, (50), and the horizontal shift is -5. Therefore our equation would look like, $D = -13\cos(\frac{\pi}{12}(t-5)) + 50$

Question 9
A population of rabbits oscillates 25 above and below an average of 129 during the year, hitting the lowest value in January ($t = 0$).
a) Find an equation for the population, P, in terms of the months since January, t.
Since this is tthe equation for a full year, the period will be 12 for 12 months. The midline was already given to us, 129, and so is the amplitude, 25. The horizontal shift is $\frac{2\pi}{12} = \frac{\pi}{6}$
Our equation will be $P(tt) = 25\cos(\frac{\pi}{6}(t)) + 129$.

Question 11, Page 449
Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature of 105 degrees occurs at 5 PM and the average temperature for the day is 85 degrees. Find the temperature, to the nearest degree, at 9 AM.
The period will be 24, the midline is 85, which was given to us. To fnd the amplitude, subtract 85 by 105 (20). The horizontal stretch factor s $\frac{2\pi}{24} = \frac{\pi}{12}$, and the horizontal shift is 17. Therefore the equation will be $P(t) = 20\cos(\frac{\pi}{12}(t)) + 85$. However the question is to find tthe temperature at 9, therefore, $P(9) = 20\cos(\frac{\pi}{12}(9-17) + 85 = 75$

Question 13
Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature varies between 47 and 63 degrees during the day and the average daily temperature first occurs at 10 AM. How many hours after midnight does the temperature first reach 51 degrees?

The midline can be found by adding 63 and 47 together and dividing by 2 (55), the amplitude can be found subtracting 47 from 63 and dividing by 2 (8), the horizontal stretch facttor is $\frac{\pi}{12}$, and tthe horizontal shift is -10. Therefore our equation is, $D(t) = 8\sin(\frac{\pi}{12}(t-10))+55$. We can find out when the temperature reached 51 by substituting 51 for t, $D(51) = 8\sin(\frac{\pi}{12}(51-10)+55$
$\sin(\frac{\pi}{12}(t-10) = 1\frac{1}{2}$
$\frac{\pi}{12}(t-10) = \sin^{-}1(-\frac{1}{2})$
$t = -\frac{\frac{\pi}{6}}{\frac{\pi}{12}} + 10 = 8$

For this section, I found it a lot easier than the previous section. I was much more in my comfort zone and was able too solve the problems without any hassle.

Overall for this chapter, I understood what was needed to be done in order to solve for the problems. However, I got very stuck on section 6.4, and I still don't know exactly why I did. I think it could have been the radians, I never have been the best when converting to radians, however I feel like other parts might have gotten me stuck as well. This chapter in my opinion has been one of the harder chapters that we've covered.